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Even-Odd Prime Harmonious Graphs

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Abstract

The paper considers finite and undirected simple connected graphs. Usually, graph labeling is an assignment of integers to the vertices or edges in the method. Naturally, Graphs arose in the study of Graham and Slaone Modular versions of additive based problems. Hence, the researcher have introduced Even-Odd Prime Harmonious Labeling of Graphs and I prove that the Path P_n where n is odd, Star $K_{1,n}$ and Comb graph $P_n.K_1$, n is even, are even-odd harmonious graphs. By the use of even-odd prime harmonious labeling concepts, one can find out the significance of even-odd prime harmonious graphs.

Keywords: Even-Odd Prime Harmonious Labeling, Prime Harmonious Labeling and Even-Odd Harmonious Labeling.

I. Introduction:

In this paper, consider finite, undirected simple connected graphs. A graph labeling is an assignment of integers to the vertices or edges or both subject to the conditions. Graph Labeling were first introduced late in the 1960's. Harmonious Graphs naturally arose in the study by Graham and Slaone by Modular versions of additive base problems. The symbols V (G) and E (G) denote the vertex set and edge set of a graph.

2. Preliminaries

We present some known definition and results related to labeling of a graph for ready reference to go through the work presented in this paper.

Definition 2.1. [1] If the values are assigned to the vertices with the certain condition(s), then it is known as graph labeling.

Definition 2.2. [6] A graph G is said to be a **harmonious graph** if there exist an injection map $f : V(G) \rightarrow \{0, 1, \ldots, 2q - 1\}$such that the induced function $f^+ : E(G) \rightarrow \{0, 1, \ldots, 2q - 1\}$ defined by $f^+(uv) = (f(u) + f(v)) \ (mod\ (q-1))$ is a bijection and f is said to be **harmonious labelling** of G.

Definition 2.3. [1] The **even-odd harmonious labeling** of a graph G with p vertices and q edges is an injection $f : V(G) \rightarrow \{1, 3, 5, \ldots, 2p + 1\}$ such that when each edge (u, v) is assigned the label $[f(u) + f(v)] \ (mod\ 2q)$ the resulting edge labels are$\{0, 2, 4, \ldots, 2(q-1)\}$. A graph which admits an even-odd harmonious labeling is called an **even-odd harmonious graph**.

Definition 2.4. [3] Let G be a graph with q edges. A function f is called prime **harmonious labeling** of a graph G if $f : V(G) \rightarrow \{0, 1, 2, \ldots, 2q - 1\}$ is injective and the induced function $f^+: E(G) \rightarrow \{0, 1, 2, \ldots, 2q\}$ defined by $f(uv) = GCD(f(u), f(v)) = 1$ and $f(e = uv) = (f(u) + f(v))\ (mod\ (q))$ is bijective. A graph which admits prime harmonious labeling is called a **prime harmonious graph.**

3. EVEN-ODD HARMONIOUS PRIME HARMONIOUS LABELING

Definition 3.1. Let G be a graph with p vertices and q edges. A function f is called an **even-odd prime harmonious labelling** of a graph G if $f: V(G) \rightarrow \{1,3,5, \ldots, 2p + 1\}$ is injective and the induced function $f^+: E(G) \rightarrow \{0,2, \ldots, 2q\}$ defined by $f^+(uv) = f(e = uv) = (f(u) + f(v))(mod(2q + 2))$ is bijective and $G.C.D(f(u), f(v)) = 1$. A graph which admits even-odd prime harmonious labeling is called an **even-odd prime harmonious graph.**

Theorem 3.2. The Path P_n where n is odd is an even-odd prime harmonious graph.

Proof.

Let $u_1, u_2, \ldots, u_n$ be the vertices of the path P_n and $e_1, e_2, \ldots, e_{n-1}$ be the edges of P_n where n is odd.

Define $f: V(P_n) \rightarrow \{1,3, \ldots, 2n - 1\}$ given by $f(u_{i+1}) = 2i + 1, 0 \le i \le n-1$

Now, $GCD(f(u_{i+1}), f(u_{i+2})) = GCD(2i + 1, 2i + 3) = 1, 0 \le i \le n - 2$

Then, f induces the edge labelling $f^+: E(P_n) \rightarrow \{0,2, \ldots, 2n\}$ by $f^+(e_{i+1}(= u_{i+1}u_{i+2})) = (2i + 1 + 2i + 3)(mod\ (2n))$ for $0 \le i \le n - 2$ which is bijective.

Hence, the Path P_n where n is odd is an even-odd prime harmonious graph.

Theorem 3.3. The Star $K_{1,n}$ is an even-odd prime harmonious graph.

Proof.

Let $u_1, u_2, \ldots, u_{n+1}$ be the vertices of the path $K_{1,n}$ and $e_1(= u_1u_2), e_2(= u_1u_3), \ldots, e_n(= u_1u_{n+1})$ be the edges of $K_{1,n}$.

Define $f: V(K_{1,n}) \rightarrow \{1,3, \ldots, 2n + 1\}$ given by $f(u_{i+1}) = 2i + 1, 0 \le i \le n - 1$

Now, $GCD(f(u_1), f(u_{i+2})) = GCD(1, 2i + 3) = 1, 0 \le i \le n - 1$

Then, f induces the edge labelling $f^+: E(K_{1,n}) \rightarrow \{0,2, \ldots, 2n\}$ by

$f^+(e_{i+1}(= u_1u_{i+2})) = (1 + 2i + 3)(mod\ (2n))$ for $0 \le i \le n - 2$ which is bijective.

Hence, the Star $K_{1,n}$ is an even-odd prime harmonious graph.

Theorem 3.4. Every Comb graphs $P_n.K_1$ where $n = 2^m$ and an even-odd prime harmonious graph is.

Proof.

Let $u_1, u_2, \ldots, u_n$ be the vertices of the path P_n in $P_n.K_1$ and let $v_1, v_2, \ldots, v_n$ be the vertices adjacent to the vertices of each vertex of the path P_n in $P_n.K_1$ and $e_1, e_2, \ldots, e_{2n-1}$ be the edges of $P_n.K_1$.

Define $f: V(P_n.K_1) \rightarrow \{1,3, \ldots, 2n - 1\}$ given by $f(u_{i+1}) = 2i + 1,\ 0 \le i \le n - 1$ and $f(v_{i+1}) = 2i + 1, 0 \le i \le n - 1$.

Now, we have $GCD(f(u_{i+1}), f(u_{i+2})) = GCD(2i + 1, 2i + 3) = 1, 0 \le i \le n - 2$ and $GCD(f(v_{i+1}), f(v_{i+2})) = GCD(2n + 2i + 1, 2n + 2i + 3) = 1, 0 \le i \le n - 2$

Then, f induces the edge labeling $f^+: E(P_n.K_1) \to \{0,2,\dots,2n\}$ by $f^+\big(e_{i+1}(=u_{i+1}u_{i+2})\big) = (2i+1+2i+3)(mod\ (4n))$for $0 \le i \le n-2$ and$f^+\big(e_{i+1}(=v_{i+1}v_{i+2})\big) = (2i+1+2n+2i+3)(mod\ (4n))$which is bijective.
Hence, the Comb graph $P_n.K_1$ where $n = 2^m$ is an even-odd prime harmonious graph.

Theorem 3.2. $C_n.K_1$ where n is odd is an even-odd prime harmonious graph.

Proof.
Let $u_1, u_2, \dots, u_n$ be the vertices of the cycle C_n in $C_n.K_1$and let u_{n+1}be the vertices adjacent to a vertex of u_n of the cycle C_n in $C_n.K_1$and $e_1, e_2, \dots, e_{n+1}$ be the edges of C_n.
Define $f: V(C_n.K_1) \to \{1,3,\dots,2n+1\}$ given by $f(u_{i+1}) = 2i+1, 0 \le i \le n$
Now, $GCD\big(f(u_{i+1}), f(u_{i+2})\big) = GCD(2i+1, 2i+3) = 1, 0 \le i \le n-1$
Then, f induces the edge labelling $f^+: E(C_n.K_1) \to \{0,2,\dots,2n\}$ by $f^+\big(e_{i+1}(=u_{i+1}u_{i+2})\big) = (2i+1+2i+3)(mod\ (2n+4))$for $0 \le i \le n-1$ which is bijective.
Hence, $C_n.K_1$where n is odd is an even-odd prime harmonious graph.

4. Conclusion

Hence, by using the even-odd prime harmonious labeling concepts, one can find out the even-odd prime harmonious graphs.

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Data Availability

The data used to support the findings of this study are from other scholarly papers for the simulation. The papers are properly cited in this article.

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